Using Plate Finite Elements for Modeling Fillets in Design, Optimization, and Dynamic Analysis

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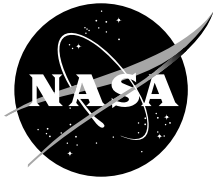
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NOMENCLATURE

A	cross-sectional area
b	base width of fillet
C	constant of integration
E	Young's modulus
E_b	Young's modulus of bridge
$E_{\text{wide-beam}}$	wide-beam Young's modulus
G	shear modulus
h_f	height of fillet
I	second moment of area
I_f	second moment of area of the fillet
$[K]_{ii}$	iith element or partition of stiffness matrix
L	length of beam plate; length of frame element
L_b	length of bridge
L_{bridge}	length of bridge element
M	moment
P	arbitrary transverse load
r	fillet radius
t	thickness of beam plate
t_b	bridge element thickness
t_f	thickness of fillet
t_{wall}	thickness of beam wall plate
v_4	transverse deflection node 4
x	location
δ	transverse displacement
θ	rotation
θ_4	rotation at node 4
θ_f	rotation of fillet
ρ_b	density of bridge
ρ_f	density of fillet
v	Poisson's ratio

TECHNICAL PUBLICATION

USING PLATE FINITE ELEMENTS FOR MODELING FILLETS IN DESIGN, OPTIMIZATION, AND DYNAMIC ANALYSIS

1. INTRODUCTION

Fillets are one of the most commonly found design features in structures. They are used for reasons varying from reducing stress concentrations to facilitating machining. In many cases, a finite element model of the structure with these fillets is required for design and analysis. For component-level modeling, either plate or solid finite elements will usually be required to give the level of fidelity desired. Upon immediate inspection, it would appear that solid modeling would be required to model the fillets accurately. Frequently, however, the most accurate and economical element for modeling the structure is the plate element. The reasons for this are that plates can more accurately predict the behavior of "plate-like" structures (defined as the ratio of the thickness to the other dimensions being <10 percent), and that the reduction in the number of degrees of freedom (DOFs) in the model is usually in excess of 90 percent. Even considering the enormous power of present day computing packages, this reduction in DOFs over solids is desirable for a variety of reasons. First, the design may require many iterations of a model or numerical optimization. Second, model creation is significantly simplified by the use of plates compared with accurately meshing fillets using solid elements; many automeshers generate unacceptable meshes of these shapes, and manual meshing is nontrivial. In addition, a smaller model significantly facilitates postprocessing. Finally, there are many situations where memory and/or speed of the computing environment is and will continue to be limited, such as in educational or small business settings.

The simplest method of using plates in these situations is simply to ignore the fillet. Obviously, the fillet stresses would not be obtained using this approach, but many types of analysis do not require a solution for detailed stresses. These include dynamic analyses, design for deflection or vibration control, or preliminary design. Clearly, though, ignoring fillets would cause an underprediction of the stiffness of the structure, especially for thin-walled configurations. Therefore, this problem provides the motivation for this research, which is to generate a method for using plate elements themselves to represent the stiffness of the fillets. This method will allow the user to take advantage of modeling with plate elements compared with the substantially more numerically intensive solid elements and to still accurately predict structural stiffness.

One example of the type of structure this method would be applicable to is the structural platform for use in a fine-motion stage[1] as shown in figure 1. The complexity of such structures leads to the obvious use of finite element methods for evaluating the performance of the design. Since a computer-aided design-based solid geometry model was created initially, the generation of a finite element mesh using solid elements was attempted using automatic meshing software. However, the intersection of thin-walled plates is problematic for these automeshers; a substantial number of tetrahedral elements with unacceptable aspect ratios are created at the fillets. In addition, the multifaceted intersections require an extremely large number of elements; design changes and subsequent analysis therefore take a considerable amount of time and effort.
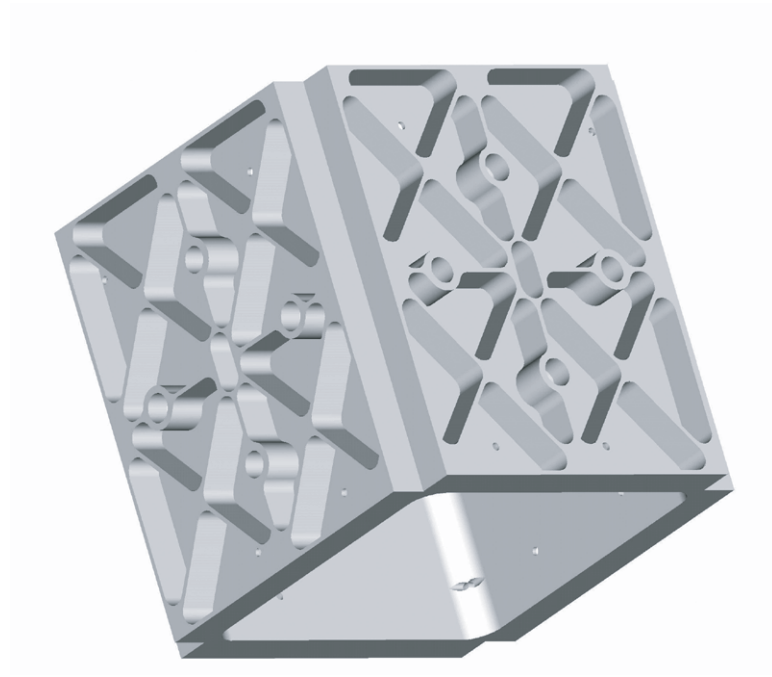
Figure 1. Four-inch cube, thin-walled structure used for 6 DOF fine motion stage, with wall thickness of 0.040 in throughout.

Based on the geometry of these fillets, various analysis methods were examined to try to obtain an accurate, yet timely, modeling methodology. Plate theory was initially investigated for the analytical basis of this study. Liu and Chang[2] and Ganesan and Nagaraja Rao[3] have performed static and vibration analysis for plates whose thickness varies. Rayleigh[4] and Timoshenko[5] have also covered vibrations of plates with uniform thickness with closed-form solutions stated for simple cases. The most extensive collection of closed-form solutions of uniform and nonuniform thickness plate theory comes from Leissa.[6] Cases of highly complex geometries are assessed experimentally to gain insight into the behavior of a complex plate constrained by simple boundary conditions. Correction factors are found for the tested geometries where strengthening ribs or webs are used and subsequently inserted back into the basic equations for simplified cases. The conclusion from this study was that the closed-form solutions for this type of analysis are limited to very simple geometries where the boundary conditions and geometry are easily defined, and that for the general fillet case, the boundary conditions prohibit strict analytic solutions.

2. THEORETICAL DEVELOPMENT

Several methods were assessed for achieving the goal of this research. The first proposed solution was to simply create a dense mesh of the plates at the filleted intersection and increase the thickness of the plates so they would match the basic shape of the fillet, as seen in figure 2. The mesh density is based on the geometry of the fillet only, and the material properties of the "fillet" plates were not altered. One weakness of this method is that as the length of the plate element decreases to allow accurate matching with the thickness of the fillet, the aspect ratio becomes unacceptable unless the depth of each element is also reduced, which would dramatically increase the mesh density and complexity of the entire model. In addition, this section of increased density and thickness plates would have to be reproduced along both in-plane axes of the fillet. Because of these weaknesses, this method was deemed to be undesirable.
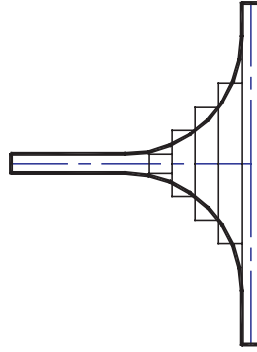

Figure 2. Method of modeling fillets by stepping
up thickness to replicate fillet geometry.

After substantial experimentation with this and various other methods, some of which are documented in previous work by the authors,[1] the procedure finally converged upon was to match the rotation at the tangent of the fillet to that of a "bridge" system. The rotation value was seen during the studies to have the most effect on accurate deflections throughout the structure. The fillet would be replaced by a system of two or three plate groups (see fig. 3), one set colinear with the nonfilleted section and a group in place of each "half" fillet section with endpoints at the fillet tangents. "Pseudo" thickness and stiffness (Young's modulus (E)) properties for the bridge groups are calculated using the derivation shown below, while the colinear section uses the same plate properties as the nonfilleted sections. This geometrical configuration is relatively simple to implement and can be used for both doubly filleted and singly filleted sections. It is, however, limited to cases where the filleted intersection forms a 90º angle. The calculated properties were seen to be a function of two parameters: the ratio of the fillet radius (r) to the thickness of the plate (t) and the ratio of the thickness of the perpendicular nonfilleted sections to the colinear nonfilleted section thickness (t_{wall}). In previous work by the authors, only the thickness of the bridge element (t_b) was generated, and this value was obtained by empirically matching the transverse stiffness of the bridge with a solid finite element model. Upon further examination and discussion, though, it was seen that the rotational stiffness

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was left out of this derivation. Implementing this stiffness was achieved by matching not just a single stiffness constant but by using the 2×2 stiffness matrix of the fillet system at the tangent node to equate the rotations at the tangent. This procedure is outlined below.
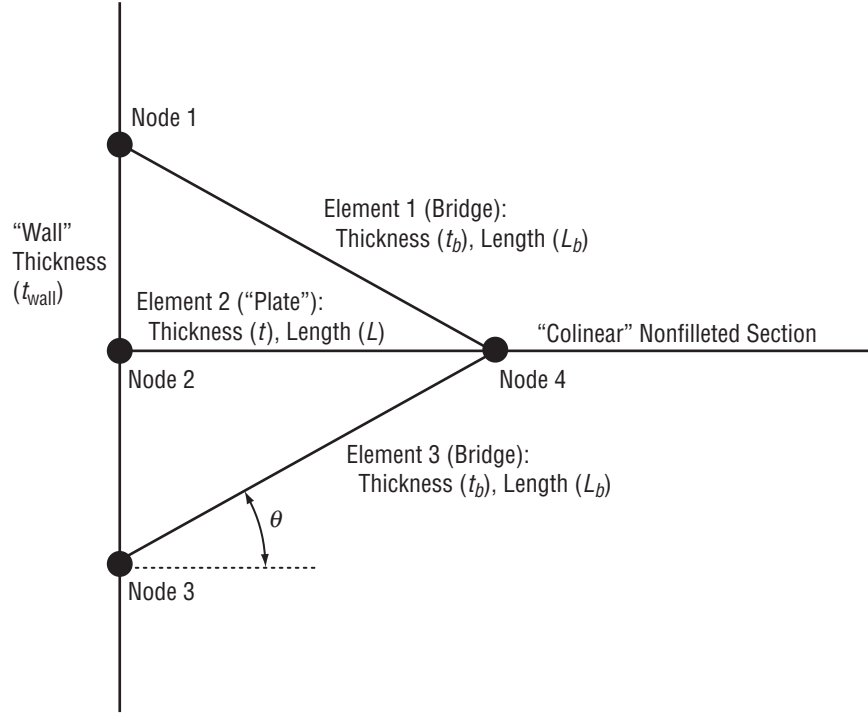

Figure 3. Representation of fillet with wide beams.

First, upon further examination of a generic fillet configuration, it is recognized that the bending stiffness of the fillet itself is only a function of its length (L), and not of its width (b). This is essentially the definition of a structure in a state of plane strain, which allows the stiffness to be reduced to in-plane transverse and rotational stiffness only. Using this assumption eliminates the out-of-plane DOFs and allows the use of "wide-beam" theory for the stiffness matrix calculations rather than plate theory,[7] which greatly simplifies the problem. This theory states that for plane-strain conditions, beam-bending equations can be used for the calculation of deflection and in-plane rotation of plates by replacing Young's modulus (E) with an adjusted value $E_{\text{wide-beam}}$, which is obtained using the equation

$$E_{\text{wide-beam}} = \frac{E}{1-v^2} \quad . \tag{1}$$

These theories were tested and verified for simple beams and plates using the finite element method, as described in section 3.

In addition, if small angles are assumed, the only contribution to the stiffness from the bridge elements is axial extension, while the only contribution from the transverse element is bending. Referring

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to figure 3, since the only DOFs of interest are the node 4 transverse deflection (v_4) and rotation (θ_4), the stiffness partitions associated with node 4 are formulated from elements 1, 2, and 3. Using the partition for a rotated plane frame element from Cook,[8] the following is obtained:

$$[K]_{44}^1, [K]_{44}^3 = \begin{vmatrix} \dfrac{AE}{L}(\sin^2\theta) + \dfrac{12EI}{L^3}(\cos^2\theta) & -\dfrac{6EI}{L^2}(\cos\theta) \\ \hline \mathrm{sym} & \dfrac{4EI}{L} \end{vmatrix}, \tag{2}$$

$$[K]_{44}^2 = \begin{vmatrix} \dfrac{12EI}{L^3} & -\dfrac{6EI}{L^2} \\ \hline \mathrm{sym} & \dfrac{4EI}{L} \end{vmatrix}. \tag{3}$$

For $\theta^1 = -45°$ and $\theta^3 = 45°$, elements 1 and 3 will yield identical results with $L_b = \sqrt{2}\,L$. It should also be noted that L is the length of the frame element, which will extend from the tangent point of the fillet to the midplane of the wall side, yielding $L = r + t_{\mathrm{wall}}/2$. If these are called "bridge" elements, and the partitions superimposed, the following is therefore obtained:

$$[K]_{44} = \begin{vmatrix} \left(\dfrac{AE}{L} + \dfrac{12EI}{L^3}\right)^{\mathrm{bridge}} + \dfrac{12EI}{L^3} & -\left(\dfrac{6\sqrt{2}\,EI}{L^2}\right)^b - \dfrac{6EI}{L^2} \\ \hline \mathrm{sym} & \left(\dfrac{8EI}{L}\right)^b + \dfrac{4EI}{L} \end{vmatrix}. \tag{4}$$

Applying the plane-strain assumption allows the use of a unit depth, which reduces the stiffness partition further to

$$[K]_{44} = \dfrac{1}{L} \begin{vmatrix} \dfrac{E_b t_b}{\sqrt{2}} + \dfrac{E_b t_b^3}{2L^2\sqrt{2}} + \dfrac{Et^3}{L^2} & -\dfrac{E_b t_b^3}{2L\sqrt{2}} - \dfrac{Et^3}{2L} \\ \hline \mathrm{sym} & \dfrac{2E_b t_b^3}{3\sqrt{2}} + \dfrac{Et^3}{3} \end{vmatrix}. \tag{5}$$

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At this point, the rotation (θ_4) of the actual fillet to any arbitrary load (P) and moment (M) at the tangent point of the fillet is determined. First, for a fillet of radius (r), the height (h_f) is related to the location (x) by

$$h_f = r - \sqrt{x(2r-x)} \ , \tag{6}$$

where x is measured from the root of the fillet. The total thickness of the fillet (t_f) is therefore

$$t_f = 2\left(\frac{t}{2} + h_f\right) , \tag{7}$$

and the second moment of area of the fillet (I_f) is defined by

$$I_f = \frac{bh_f^3}{12} \ , \tag{8}$$

where the base (b) is equal to the unit depth 1. The rotation $\theta_f(x)$ can therefore be expressed as

$$\theta_f = \int \frac{(M + P \times r)}{EI_f} dx + C \ . \tag{9}$$

Since the boundary condition on the root side of the fillet is zero rotation; i.e., v'(x=0)=0, the constant of integration C will equal the negative of the value obtained from the indefinite integral. The symbolic integration is performed using Mathematica™ 4.1 but is not included in this Technical Publication. θ_4 is determined by setting x=r in equation (9).

Similar expressions for the rotation at node 4 are now obtained for the bridge system. These are obtained by multiplying the inverse of the stiffness partition $[K]_{44}$ by a column vector of the arbitrary moment and load. A solution is being sought such that the rotation is the same for the bridge system as the actual fillet, so the solutions are equated for this value from each system. Since the solutions have to be valid for any value of M or P, the coefficients for each of these parameters can therefore be collected and equated, leaving two nonlinear equations for the unknowns E_b and t_b:

$$\frac{\dfrac{Et^3}{\left(r+\frac{t_{\text{wall}}}{2}\right)^2} + \dfrac{E_b\, t_b^{\,3}}{2\sqrt{2}\left(r+\frac{t_{\text{wall}}}{2}\right)^2} + \dfrac{E_b\, t_b}{\sqrt{2}}}{\left[\dfrac{E^2 t^6}{12\left(r+\frac{t_{\text{wall}}}{2}\right)^4} + \dfrac{E\,E_b\, t_b^{\,3}\, t^3}{3\sqrt{2}\left(r+\frac{t_{\text{wall}}}{2}\right)^4} + \dfrac{E\,E_b\, t_b\, t^3}{3\sqrt{2}\left(r+\frac{t_{\text{wall}}}{2}\right)^2} + \dfrac{E_b^{\,2}\, t_b^{\,6}}{24\left(r+\frac{t_{\text{wall}}}{2}\right)^4} + \dfrac{E_b^{\,2}\, t_b^{\,4}}{3\left(r+\frac{t_{\text{wall}}}{2}\right)^2}\right]\left(r+\frac{t_{\text{wall}}}{2}\right)}$$

$$= \frac{6\left(8r^2\sqrt{4r+t}\;t^{3/2} + 2r\sqrt{4r+t}\;t^{5/2} + 12r^3\sqrt{4r+t}\;\sqrt{t} + 3\pi r^2(2r+t)^2 + 6r^2(2r+t)^2\tan^{-1}\!\left(\dfrac{2r}{\sqrt{t}\sqrt{4r+t}}\right)\right)}{Et^{5/2}(2r+t)\left(4r+t\right)^{5/2}}\;; \quad (10a)$$

$$\frac{\dfrac{E\, t^3}{2\left(r+\frac{t_{\text{wall}}}{2}\right)} + \dfrac{E_b\, t_b^{\,3}}{2\sqrt{2}\left(r+\frac{t_{\text{wall}}}{2}\right)}}{\left[\dfrac{E^2 t^6}{12\left(r+\frac{t_{\text{wall}}}{2}\right)^4} + \dfrac{E\,E_b\, t_b^{\,3}\, t^3}{3\sqrt{2}\left(r+\frac{t_{\text{wall}}}{2}\right)^4} + \dfrac{E\,E_b\, t_b\, t^3}{3\sqrt{2}\left(r+\frac{t_{\text{wall}}}{2}\right)^2} + \dfrac{E_b^{\,2}\, t_b^{\,6}}{24\left(r+\frac{t_{\text{wall}}}{2}\right)^4} + \dfrac{E_b^{\,2}\, t_b^{\,4}}{3\left(r+\frac{t_{\text{wall}}}{2}\right)^2}\right]\left(r+\frac{t_{\text{wall}}}{2}\right)}$$

$$= \frac{6\left(4\left(r^2\right)^{3/2}\sqrt{4r+t}\;t^{3/2} + 4r^3\sqrt{4r+t}\;t^{3/2} + r^2\sqrt{4r+t}\;t^{5/2} + 8r^4\sqrt{4r+t}\;\sqrt{t} + 8r^3\sqrt{r^2}\sqrt{4r+t}\sqrt{t}\right)}{Et^{5/2}(2r+t)\left(4r+t\right)^{5/2}}\,. \quad (10b)$$

Using Mathematica 4.1, this can then be solved symbolically for the desired parameters E_b and t_b in terms of r and t_{wall}. The solution was verified by substituting the values back into the bridge and fillet system rotation equations to check for equivalence. The solution is readily obtained using Mathematica, but for

ease of use, results for the most applicable range of parameters have been tabularized (tables 1 and 2) as well as fit to an explicit equation. The tables can be imported as matrices into the program Matlab™, where the "surfht(twall, r, eb)" command will yield an interactive interpolated grid where the E_b/E or t_b/t value can be easily obtained. The surface fit of these data is shown in equations (11) and (12) where $x=r/t$ and $y=t_{\text{wall}}/t$. The maximum errors of the equations from the data are 2.8 percent, and are generally <1 percent:

$$\frac{E_b}{E} = \frac{0.9782601242110346}{x^2} - \frac{0.7149708253246347}{x^3} - \frac{1.9678245005077897}{x^4} + \frac{1.4899111209264795}{x^5}$$

$$+ \frac{0.045579223088219975}{y} + \frac{0.7522289111879881}{xy} + \frac{2.088608970319005}{x^2 y} + \frac{3.898702480893012}{x^3 y}$$

$$- \frac{0.34762995889834863}{y^2} - \frac{1.5284136218083295}{xy^2} - \frac{1.5528800000806626}{x^2 y^2} + \frac{0.784033948805702}{y^3}$$

$$+ \frac{0.8445177307299556}{xy^3} - \frac{0.6771296045396015}{y^4} + \frac{0.19714890087642176}{y^5} - \frac{0.08229688966201235}{x} \qquad (11)$$

$$\frac{t_b}{t} = -0.000704997\, x^2 - 0.00390799\, yx + 0.316686\, x - 0.00362814\, y^2 + 0.200802\, y - 0.297332 \ . \quad (12)$$

Table 1. E_b/E normalized bridge Young's modulus.

$(r/t)\backslash tw/_t$	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	3.667	4.000	4.333	4.667	5.000
2	0.6952	0.6092	0.5439	0.4922	0.4501	0.4150	0.3854	0.3599	0.3377	0.3183	0.3012	0.2859	0.2721
3	0.2645	0.2446	0.2276	0.2130	0.2002	0.1889	0.1789	0.1699	0.1619	0.1546	0.1480	0.1419	0.1364
4	0.1368	0.1294	0.1228	0.1169	0.1116	0.1067	0.1023	0.0982	0.0945	0.0910	0.0878	0.0849	0.0821
5	0.0830	0.0795	0.0763	0.0734	0.0707	0.0682	0.0659	0.0637	0.0617	0.0598	0.0581	0.0564	0.0549
6	0.0555	0.0536	0.0519	0.0502	0.0487	0.0472	0.0459	0.0446	0.0434	0.0423	0.0412	0.0402	0.0392
7	0.0397	0.0385	0.0375	0.0365	0.0355	0.0346	0.0337	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
8	0.0297	0.0290	0.0283	0.0276	0.0270	0.0264	0.0258	0.0253	0.0248	0.0243	0.0238	0.0233	0.0229
9	0.0231	0.0226	0.0221	0.0217	0.0212	0.0208	0.0204	0.0200	0.0197	0.0193	0.0190	0.0186	0.0183
10	0.0185	0.0181	0.0178	0.0174	0.0171	0.0168	0.0165	0.0162	0.0160	0.0157	0.0155	0.0152	0.0150

Table 2. t_b/t normalized bridge thickness.

$(r/t)\backslash tw/t$	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	3.667	4.000	4.333	4.667	5.000
2	0.5084	0.5795	0.6469	0.7112	0.7730	0.8325	0.8900	0.9458	1.0000	1.0528	1.1043	1.1545	1.2036
3	0.8216	0.8858	0.9481	1.0087	1.0677	1.1253	1.1815	1.2365	1.2904	1.3432	1.3950	1.4458	1.4958
4	1.1357	1.1958	1.2548	1.3125	1.3692	1.4248	1.4794	1.5331	1.5859	1.6378	1.6890	1.7393	1.7890
5	1.4474	1.5048	1.5612	1.6167	1.6714	1.7253	1.7783	1.8307	1.8823	1.9332	1.9834	2.0330	2.0820
6	1.7561	1.8112	1.8656	1.9193	1.9723	2.0246	2.0763	2.1274	2.1778	2.2277	2.2770	2.3258	2.3740
7	2.0613	2.1147	2.1674	2.2195	2.2711	2.3220	2.3725	2.4223	2.4717	2.5206	2.5690	2.6169	2.6643
8	2.3631	2.4149	2.4662	2.5170	2.5672	2.6170	2.6663	2.7151	2.7635	2.8114	2.8590	2.9060	2.9527
9	2.6616	2.7121	2.7621	2.8116	2.8607	2.9094	2.9577	3.0056	3.0530	3.1001	3.1468	3.1931	3.2390
10	2.9568	3.0061	3.0550	3.1035	3.1516	3.1993	3.2466	3.2935	3.3401	3.3864	3.4323	3.4779	3.5231

Finally, for dynamic applications, it is also important to accurately represent the mass of the original fillet. This is accomplished by deriving a pseudo mass (ρ_b) that can be easily assigned to the bridge elements. This equation is obtained as a function of the actual density of the fillet (ρ_f), assuming the use of a homogenous material throughout the fillet, and by matching the volume of the bridge elements with the volume of the original fillet:

$$\rho_b = \frac{t\, 4\sqrt{2}}{r(4-\pi)}\rho_f \quad .$$

(13)

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3. VALIDATION

Several levels of validation are required for verification of the technique described. The first is to verify the plane-strain assumption of a transversely loaded fillet. This assumption was verified by building a high-fidelity solid model of a cantilevered 1-in- (2.54-cm-) wide filleted section (radius of 0.25 in (0.635 cm), thickness of 0.04 in (0.1016 cm)), loading it transversely with a 1-lb (4.448-N) static load, and comparing the directional strains at a node at the tangent of the fillet. The results are in-plane strains of 501 μstrain and 243 μstrain, and an out-of-plane strain of 1 μstrain, verifying the assumption. The capability of wide-beam theory to represent plane-strain configurations was then examined. For a cantilevered plate with a 10-in (25.4-cm) length, 100-in (254-cm) width, 1-in (2.54-cm) thickness, with a 1-lb/in (1.75-N/cm) end load, the theoretical value of displacement (δ) is 3.8037×10^{-4} in (9.6614×10^{-4} cm) using the "wide-beam" adjusted Young's modulus where

$$\delta = \frac{PL^3}{3EI} + \frac{6(PL)}{5(AG)} \ . \tag{14}$$

A high-fidelity plane-strain MSC/NASTRAN finite element model yielded the exact same result.

The next validation step is to verify the validity of using beam theory to generate the fillet stiffness. For the following parameters,

$$E = 10.6 \times 10^6 \text{ psi (73.084 GPa)}, \ \ r = 0.25 \text{ in (0.635 cm)}, \ \ t = 0.064 \text{ in (1.626 cm)}, \ \ \nu = 0.33,$$
$$P = 1\tfrac{\text{lb}}{\text{in}} \text{ (6.894 kPa)}, \ \ M = 1.867 \text{ lb-in (21.093 Nt-cm)}, \ \ \text{and } t_{\text{wall}} = 0.064 \text{ in (1.626 cm)} \ , \tag{15}$$

the rotation at the tangent point was calculated using the fillet wide-beam theory outlined above to be 5.3134×10^{-4} rd. For comparison, a high-fidelity plane-strain finite element model (fig. 4) using the same parameters resulted in a rotation at the tangent of 5.76156×10^{-4} rd, an error of 7.78 percent. This error is larger than anticipated, but is acceptable for use in the methodology developed here.

Finally, a comprehensive test was performed by applying the technique on a representative structure that could be modeled with a high-fidelity solid mesh and that could be constructed and tested. A 5×2.5×1.26 in (12.7×6.35×3.2 cm) double "box" structure was machined and modeled to meet these criteria. A photo of the structure is shown in figure 5, and the highly dense solid mesh and plate mesh using the bridge system are shown in figures 6 and 7. To assess the impact of the fillet, the structure was also modeled using plates but with the fillets completely ignored.
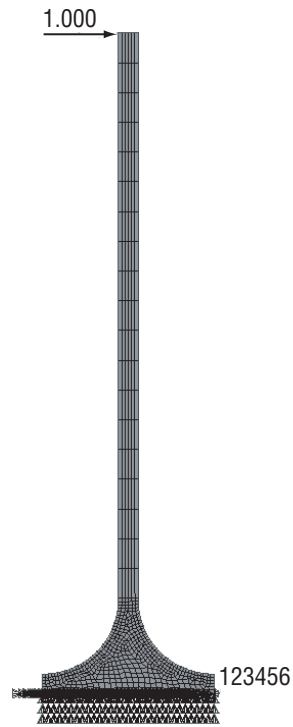
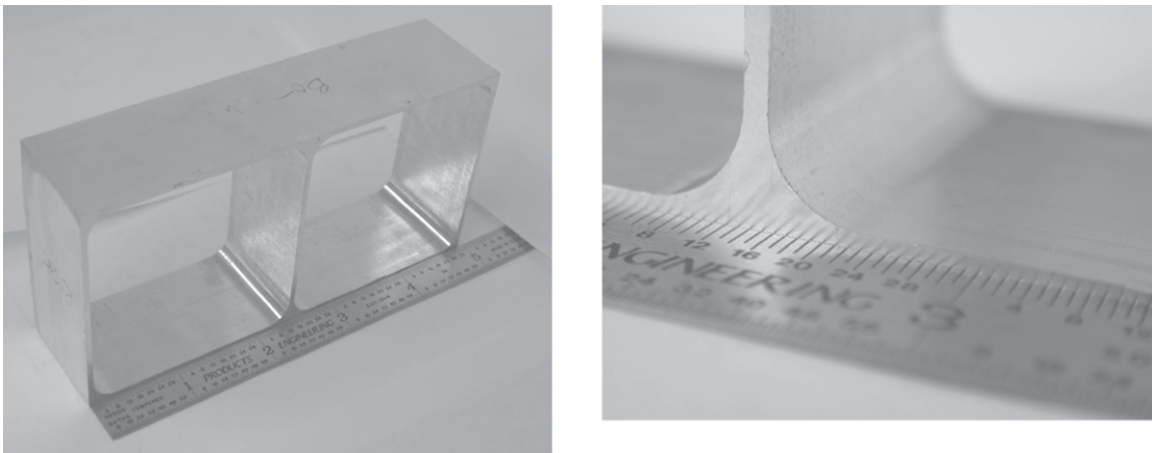Figure 4. Fillet high-fidelity plane-strain finite element model.

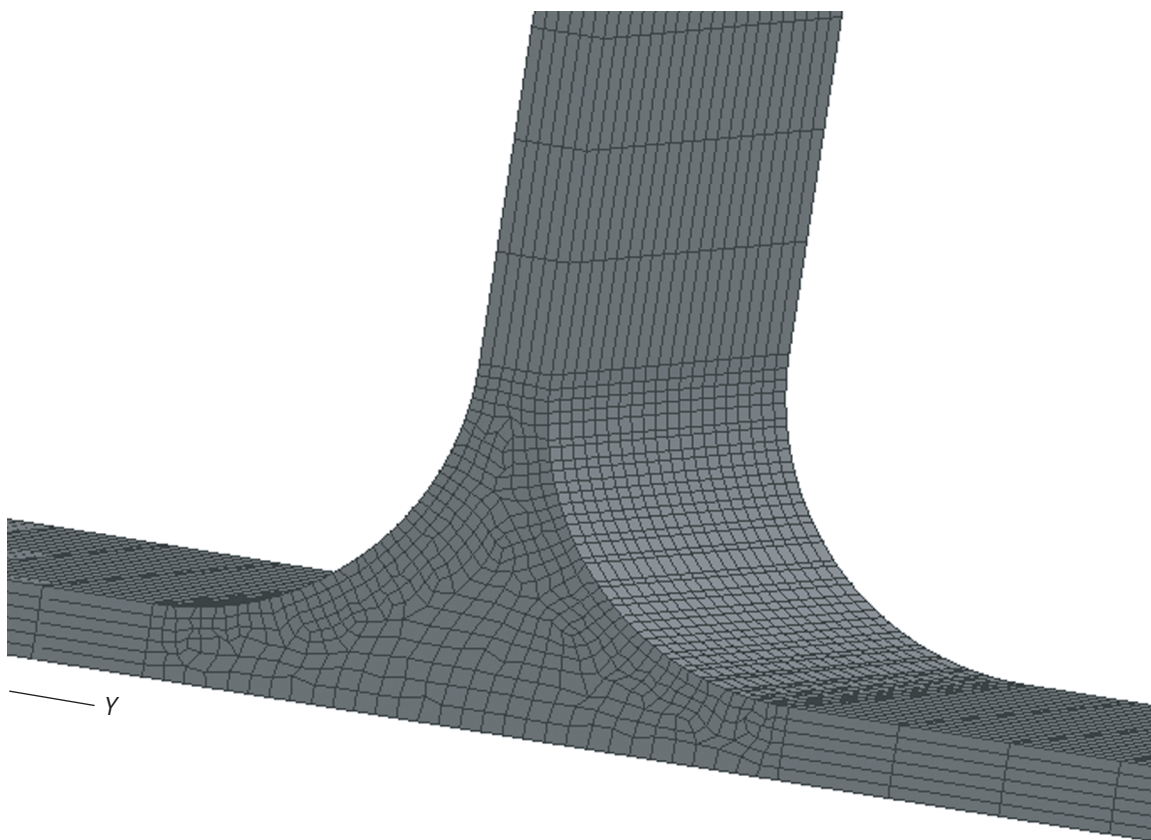Figure 5. Typical filleted thin-walled structure.
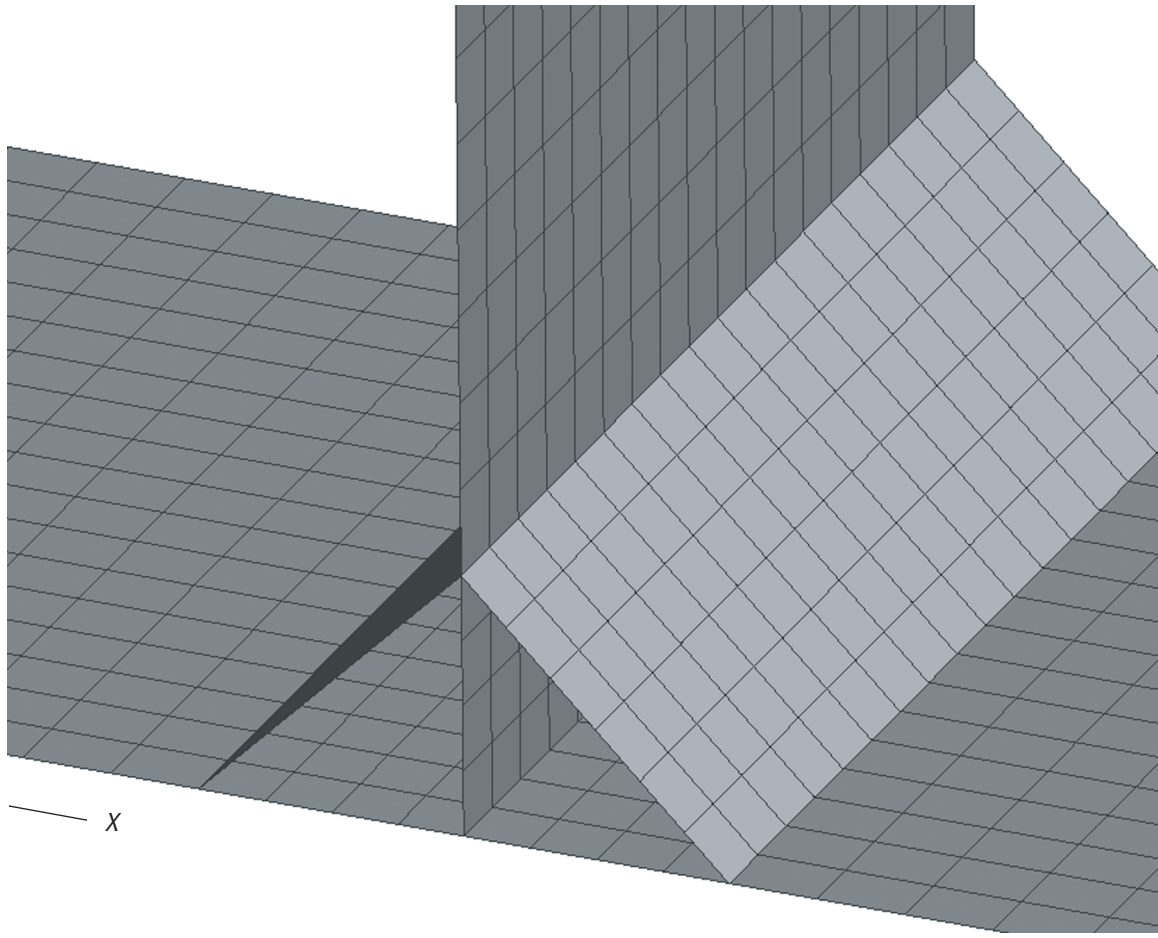
Figure 6. Detail of mesh of solid model.

Figure 7. Detail of fillet for "bridge" plate model.

Comparisons of the size of the models and results of a modal analysis and test are shown in table 3. Mode 8 for the two models is shown in figures 8 and 9. The solid model has 51,520 nodes (257,600 DOFs) and 45,120 elements, while the plate "bridge" element model has 4,465 nodes (22,325 DOFs) and 4,392 plate elements. The results show that the application of the technique results in at least a 90-percent reduction in the number of DOFs with no loss of accuracy, and that ignoring the fillets underpredicts the natural frequencies substantially.

Table 3. Comparison of models and test for double box.

| Free-Free Mode Number	Test Frequency (Baseline)	Solid Model	Frequency Error (%)	Plate Element Model With "Bridge" Fillets	Error (%)	Plate Element Model Without Fillets	Error (%)
7	484.0	487.6	0.8	494.6	2.2	449.1	−7.2
8	540.0	534.3	−1.1	541.0	0.2	467.6	−13.4
9	632.0	627.4	−0.7	633.8	0.3	547.1	−12.8
10	1,012.0	999.7	−1.2	998.7	−1.3	1,000.7	−1.1
11	1,412.0	1,403.4	−0.6	1,408.8	−0.2	1,282.2	−9.2
12	1,900.0	1,903.6	0.2	1,921.8	1.1	1,687.7	−11.2
13	2,320.0	2,290.8	−1.3	2,322.4	0.1	1,992.3	−14.1
14	2,810.0	2,792.6	−0.6	2,806.6	−0.1	2,291.4	−18.5
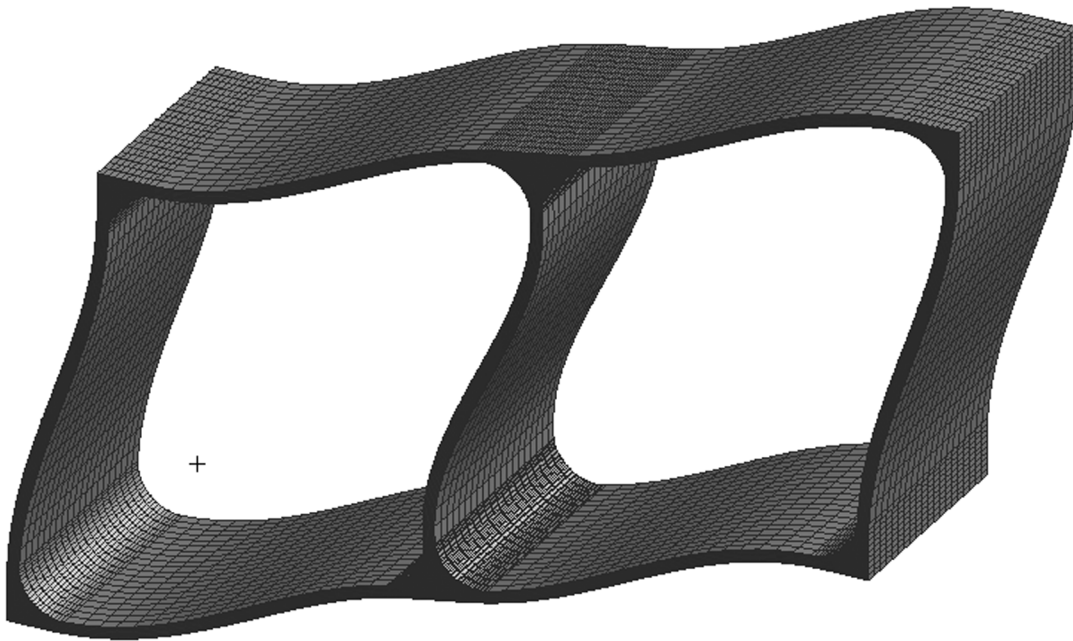
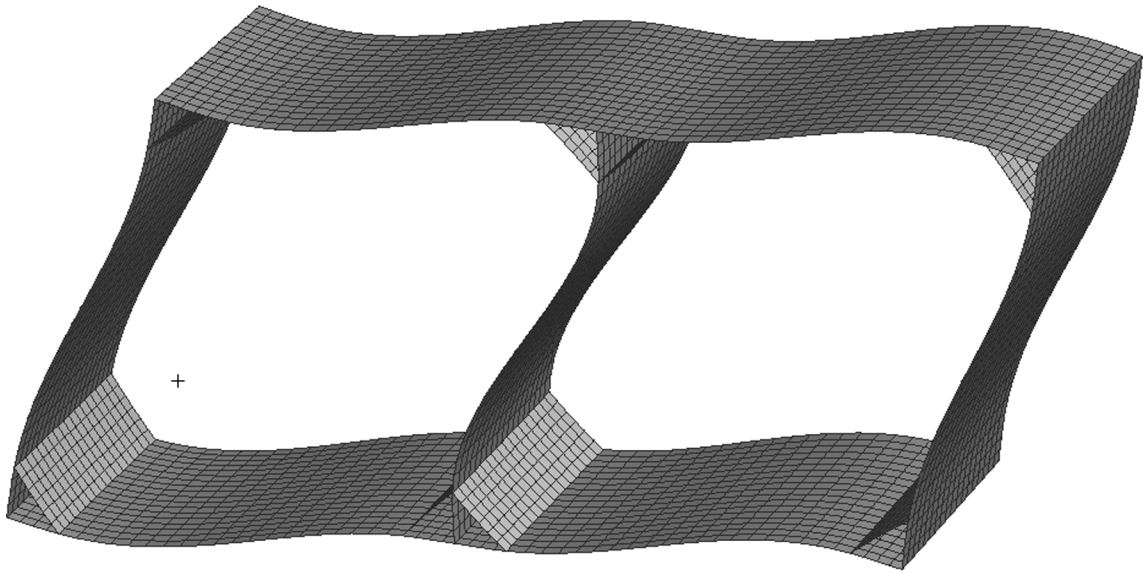Figure 8. Solid model, free-free mode 8 at 524.5 Hz.

Figure 9. "Bridge" plate element mode 8 at 541 Hz.

4. CONCLUSIONS AND FUTURE WORK

A methodology has been developed that allows the use of plate elements instead of numerically inefficient solid elements for modeling structures with 90° fillets. The technique uses plate "bridges" with pseudo Young's modulus (E_b) and thickness (t_b) values placed between the tangent points of the fillets. These parameters are obtained by solving two nonlinear simultaneous equations in terms of the independent variables r/t and t_{wall}/t. These equations are generated by equating the rotation at the tangent point of a bridge system with that of a fillet, where both rotations are derived using beam theory. Accurate surface fits of the solutions are also presented to provide the user with closed-form equations for the parameters. The methodology was verified on the subcomponent level and with a representative filleted structure, where the technique yielded a plate model exhibiting a level of accuracy better than or equal to a high-fidelity solid model and with a 90-percent reduction in the number of DOFs. The application of this method for parametric design studies, optimization, and dynamic analysis should prove extremely beneficial for the finite element practitioner. Although the method does not attempt to produce accurate stresses in the filleted region, it can also be used to obtain stresses elsewhere in the structure for preliminary analysis. A future avenue of study is to extend the theory developed here to other fillet geometries, including fillet angles other than 90° and multifaceted intersections.

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REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY *(Leave Blank)*	2. REPORT DATE March 2003	3. REPORT TYPE AND DATES COVERED Technical Publication

4. TITLE AND SUBTITLE

Using Plate Finite Elements for Modeling Fillets in Design, Optimization, and Dynamic Analysis

5. FUNDING NUMBERS

6. AUTHORS

A.M. Brown and R.M. Seugling*

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES)

George C. Marshall Space Flight Center
Marshall Space Flight Center, AL 35812

8. PERFORMING ORGANIZATION REPORT NUMBER

M–1068

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, DC 20546–0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA/TP—2003–212340

11. SUPPLEMENTARY NOTES

*The University of North Carolina at Charlotte, 9201 University City Blvd., Charlotte, NC 28223
Structures, Mechanics, and Thermal Department, Engineering Directorate

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified-Unlimited
Subject Category 39
Standard Distribution

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Fillets are one of the most common design features in structures. Proper finite element modeling of these fillets can frequently be problematic though. If the ratio of the fillet radius to the wall thickness is relatively large, the fillet cannot be ignored because it contributes significantly to structural stiffness, and although the most appropriate element for modeling the structure in general may be the plate element, geometric representation of the fillets requires the use of solid elements. This problem is the motivation for the development of a method that uses "bridge" plate elements connecting the tangent points of the fillet to accurately represent its stiffness and mass. The methodology equates the rotational deflection at the tangent point, derived from the proposed bridge system, with an analytical solution of the fillet itself to generate a pseudo Young's Modulus and thickness for use in the bridge plates. The method was tested on a typical filleted structure, with the bridge method yielding modal analysis results as accurate as a high-fidelity solid model when compared to modal test but with a 90-percent reduction in number of degrees of freedom. This capability could prove extremely useful in design, dynamic, deflection, and preliminary stress analysis, and optimization.

14. SUBJECT TERMS

finite element analysis, fillets, thin-walled structures, structural analysis, optimization, variable cross section

15. NUMBER OF PAGES
24

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT Unlimited

National Aeronautics and
Space Administration
AD33
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama
35812